Robustness Improvement of Actively Controlled Structures through Structural Modifications

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The parameter variations introduced by the analysis model, uncertain material properties, or optimization may adversely influence the stability and performance characteristics of a closed-loop controlled structure. The improvement of robustness of actively controlled structures through structural modifications is considered in this work. The stability and performance robustness indices are defined as measures of robustness of actively controlled structures. The integrated structural/control design problem is considered as a multiobjective optimization problem, in which three objectives—structural weight, stability robustness index, and performance robustness index—are considered for minimization. The utility function, lexicographic, and goal programming methods are applied to solve the multiobjective nonlinear programming problem. Two examples, a two-bar truss and two-bay truss, are considered to demonstrate the procedure.

Introduction

THERE has been a dramatic increase in the past decade in the use of active control systems to improve structural performance.[1,2] The major challenge in the field of active control of structures is in the design of control systems for very large space structures. These structures are by nature distributed parameter systems with multiple inputs (controls) and a continuum of outputs (displacements). The finite-element method is commonly used for the description of these structures. This is a source of parameter errors and truncated (or reduced order) models in the system. In addition, the structural properties of large space structures cannot be tested before they are put into orbit and, hence, sizeable uncertainties exist in modal parameters.

A great deal of research is currently in progress on developing methods for the simultaneous (integrated) design of the structure and the control system. The weight of the structure was minimized, with constraints on the distribution of the eigenvalues and/or damping ratio of the closed-loop system by Khot et al.[3] Miller and Shim[4] considered the simultaneous minimization, in structural and control variables, of the sum of structural weight and the infinite horizon linear regulator quadratic control cost. The structure/control system optimization problem was formulated by Khot et al.,[5] with constraints on the closed-loop eigenvalue distribution and the minimum Frobenious norm of the control gains. It can be seen that in all the above works the consideration of robustness of the control system has been ignored.

The parameter variations introduced by the analysis model, uncertain material properties, or optimization may adversely influence the stability and performance characteristics of the control system. The robustness is an extremely important feature of a feedback control design. A robust control design is one that satisfactorily meets the system specifications, even in the presence of parameter uncertainties and other modeling errors. Since the system specifications could be in terms of stability and/or performance, two types of robustness, namely, stability robustness and performance robustness, are to be considered in the design stage.

The current published literature on control system robustness addresses either the stability robustness aspect or the performance robustness aspect. Most of the work on the stability robustness (in the control area) was done in the frequency domain, using singular value decomposition; much of the useful research on performance robustness was carried out in time domain, using sensitivity approaches. Design studies that treated the stability robustness aspect in time domain and studies that combined both stability robustness and performance robustness in the design process have been scarce. The recent development in the area of robust multivariable control theory have been summarized by Ridgely and Banda.[6] The stability robustness of linear systems was analyzed in the time domain in Ref. 7, wherein a bound on the perturbation of an asymptotically stable linear system was obtained to maintain stability using a Lyapunov matrix equation solution. In Ref. 8, singular value robustness measures were used to compare the performance and stability robustness properties of different control design techniques in the presence of residual modal interaction for a flexible spacecraft system. The importance of robustness considerations in the design of flexible space structures was discussed by Hoehne.[9] Gordon and Collins[10] presented a direct design method for solving the problem of robustness to cross-coupling perturbations by treating the feedback gains as design variables. Their method makes use of nonlinear programming techniques along with a time domain pole placement procedure. A technique for the improvement of stability robustness by shaping the singular value spectrum, using constrained optimization methods, was described in Ref. 11.

The following problems are considered in this work: 1) effect of change in structural parameters on the robustness of structures; 2) effect of structural optimization on the robustness of the control system design; and 3) multiobjective optimization of actively controlled structure by treating the structural weight, the stability robustness index, and the performance robustness index as the objectives for minimization.

System Formulation and Basic Equations

The equations of motion of a large space structure with active controls under external forces can be expressed in state

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space form as

$$x = [A]x + [B]F \tag{1}$$

where $[A]$ is the $n \times n$ plant matrix and $[B]$ is the $n \times p$ input matrix given by

$$[A] = \begin{bmatrix} [O] & [I] \\ -[\bar{K}] & -[\bar{C}] \end{bmatrix} \tag{2}$$

$$[B] = \begin{bmatrix} [O] \\ [\phi]^T & [D] \end{bmatrix} \tag{3}$$

$$[\bar{M}] = [\Phi]^T[M][\Phi] = [I] \tag{4}$$

$$[\bar{C}] = [\Phi]^T[C][\Phi] = \mathrm{diag}[2\xi_i\omega_i] \tag{5}$$

$$[\bar{K}] = [\Phi]^T[K][\Phi] = \mathrm{diag}[\omega_i^2] \tag{6}$$

where ξ_i is the damping ratio of the ith mode and ω_i is the natural frequency of the ith mode. The state output equation is given by

$$y = [\underset{\sim}{C}]x \tag{7}$$

where y is the $q \times 1$ output vector and $[C]$ is the $q \times n$ output matrix. If the sensors and actuators are colocated, then $q = p$ and

$$[\underset{\sim}{C}] = [B]^T \tag{8}$$

In order to design a controller using a linear quadratic regulator, a performance index j is defined as

$$J = \int_0^\infty (y^T[Q]y + F^T[R]F)\, \mathrm{d}t \tag{9}$$

where $[Q]$ is the state weighting matrix that has to be positive semidefinite and $[R]$ is the control weighting matrix that has to be positive definite. Equations (1–3) and Eq. (7) are assumed to correspond to the system with nominal design variables. For this system, an optimal state feedback controller is designed by linear quadratic regulator, as

$$F = -[R]^{-1}[B]^T[K]x \equiv -[G]x \tag{10}$$

where $[K]$ satisfies the algebraic Riccati equation:

$$0 = [A]^T[K] + [K][A] - [K][B][R]^{-1}[B]^T[K] + [\underset{\sim}{C}]^T[Q][\underset{\sim}{C}] \tag{11}$$

This controller minimizes the quadratic performance index J. If there is a bounded uncertainty or disturbance in the design variables, then the plant matrix $[A]$, input matrix $[B]$, and the output matrix $[\underset{\sim}{C}]$ will be changed to $[\tilde{A}]$, $[\tilde{B}]$, and $[\tilde{C}]$, respectively. If the controller designed for the nominal system, given by Eq. (10), is used for this case, it might cause the system to be unstable. For this reason, it is necessary to consider stability robustness.

Robustness Analysis and Problem Formulation

In this work, the optimal control law is used to design the controller of the structure. Under the permissible uncertainty in the design variables, the stability robustness is maximized. For this, a stability robustness index is defined as the change in the dominant eigenvalue of the closed-loop system, as

$$\beta_{sr} = \frac{\left||\lambda_1| - |\tilde{\lambda}_1|\right|}{|\lambda_1|} \tag{12}$$

where λ_1 is the dominant eigenvalue of the system $([A] - [B][G])$, and $\tilde{\lambda}_1$ is the dominant eigenvalue of the system $([\tilde{A}] - [\tilde{B}][G])$. Note that $Re(\lambda_i) < 0$ if and only if the original system is asymptotically stable and $Re(\tilde{\lambda}_i)$ is not guaranteed to be negative. Hence, the stability requirement, $Re(\tilde{\lambda}_i) < 0$, is used as a constraint in the optimization procedure. In addition to the stability robustness, it is desirable to retain the performance unchanged when the design variables change. Since the performance cannot remain the same, a performance robustness index is defined, as follows. From Eq. (9), the steady-state solution of the stable system, as $t_f \to \infty$, gives

$$J = x_o^T[K]x_o \tag{13}$$

where x_o denotes the initial state variable vector. For the modified system with $[\tilde{A}]$, $[\tilde{B}]$, and $[\tilde{C}]$

$$\tilde{J} = x_o^T[\tilde{K}]x_o \tag{14}$$

where $[\tilde{K}]$ satisfies the Lyapunov equation

$$0 = [\tilde{A}_{cl}]^T[\tilde{K}] + [\tilde{K}][\tilde{A}_{cl}] + ([G]^T[R][G] + [\tilde{C}]^T[Q][\tilde{C}]) \tag{15}$$

The performance robustness index β_{pr} is defined as

$$\beta_{\mathrm{pr}} = \left|\frac{J - \tilde{J}}{J}\right| = \frac{x_o^T(K - \tilde{K})x_o}{x_o^T K x_o} \tag{16†}$$

It has been pointed out in Ref. 12 that the optimal solution of Eq. (16), in general, depends on the initial state x_o. This result is not very useful since the initial state is not always known. In Ref. 12, the effect of x_o is averaged out by assuming that x_o is a uniformly distributed random vector whose covariance is given by the identity matrix. It can be shown that the trace of K is proportional to the expected value of J. Hence, the performance robustness index is redefined as

$$\beta_{pr} = \left|\frac{Tr(K) - Tr(\tilde{K})}{Tr(K)}\right| \tag{17}$$

For both nominal and perturbed systems, good dynamic response can be achieved if the real part of every eigenvalue is restricted to be smaller than a specified value, as $Re(\lambda) < -a$, with $a > 0$. The LQ regulator can be modified[13] as follows, with the requirement of $([A],[B])$ being controllable and $([A],[C])$ being observable:

$$\dot{x} = (A + aI)x + Bu = (A + aI - BG)x \tag{18}$$

$$u = -R^{-1}B^TK_ax = -Gx \tag{19}$$

with K_a satisfying the equation

$$0 = (A + aI)^TK_a + K_a(A + aI) - K_aBR$$
$$-1B^TK_a + C^TQC \tag{20}$$

The characteristic equation of Eq. (18) is given by

$$\det[(A + aI - BG) - \lambda I] = \det[A - BG - \lambda' I] = 0 \tag{21}$$

with $\lambda' = \lambda - a$. Since A is assumed to be stable, then $Re(\lambda') < 0$ and, hence, $Re(\lambda) < -a$. Thus, the system of Eqs. (1–3) and Eq. (8), along with the controller of Eq. (19), provide a guaranteed stability margin.

Multiobjective Design Problem

The stability robustness, the performance robustness, and the total structural weight are considered as the objective functions in this work. The cross-sectional areas of the members are treated as the design variables. The first objective function, the stability robustness index (β_{sr}), describes the relative stability of the system when the design variables change by a specified amount. It is assumed that the con-

†For simplicity, brackets are not used to denote matrices, hereafter.

troller gains are such that the condition for the stability of the system is satisfied and, thus, the closed-loop system matrix of the perturbed system is still stable. According to this definition, $\beta_{sr} = 0$ corresponds to highly robust system from the stability point of view. However, β_{sr} will not attain the value zero due to the presence of perturbations in the design variables. The second objective function, the performance robustness index of the system (β_{pr}), is defined by Eq. (17). Here also, $\beta_{pr} = 0$ corresponds to a highly robust system from the performance point of view. The value $\beta_{pr} = 0$ will not be attained in practice due to the presence of perturbations in the design variables. The third design objective function, the total structural weight, is given by

$$f_3(x) = \sum_{i=1}^{N} \rho_i l_i A_i \tag{22}$$

with ρ_i, l_i, and A_i denoting the density, length, and cross-sectional area of the ith member, respectively, and N representing the number of members in the truss structure. The following constraints are used during the optimization procedure:

1) Upper and lower bounds on the design variables.
2) Stability requirement, i.e., the requirement of the real parts of the eigenvalues of the closed-loop system to be negative,
3) Lower and/or upper bounds on the natural frequencies of vibration of the structure.

In some cases, the closed-loop damping ratios of the system may have to be constrained; however, these are not considered in this work.

Multiobjective Optimization

The three-objective optimization problem formulated is solved using the utility function, the lexicographic, and the goal programming methods. The utility function method involves the solution of the following problem:[14]

$$\text{Min}\, U(x) = \sum_{i=1}^{k} w_i f_i(x)$$

subject to

$$g_j(x) \leq 0, \qquad j = 1, 2, \ldots, m \tag{23}$$

where w_i is the weight of the ith objective function f_i and $\Sigma_{i=1}^{k} w_i = 1$. Usually, the scales and units of different objective functions are different. Hence, a suitable normalization process has to be used in constructing the objective functions of Eq. (23). A convenient form is to define a new objective function F_i as

$$F_i(x) = \frac{f_i(x) - f_i^*(x^*)}{f_i^*(x^*)} \tag{24}$$

and redefine the objective function of Eq. (23) as

$$\text{Min}\, U(x) = \sum_{i=1}^{k} w_i F_i(x) \tag{25}$$

In the lexicographic method, the objectives are ranked in order of importance by the designer. The preferred solution obtained by this method is one that minimizes the objectives, starting with the most important one and proceeding according to the order of importance of the objectives. Let the subscripts of the objectives indicate not only the objective function number, but also the priorities of the objectives. Thus, $F_1(x)$ and $F_k(x)$ denote the most and the least important objective functions, respectively. Then the first problem is formulated as

$$\text{Min}\, F_1(x)$$

subject to

$$g_j(x) \leq 0, \qquad j = 1, 2, \ldots, m \tag{26}$$

and its solutions x_1^* and $F_1^* = F_1(x_1^*)$ are obtained. Then, the second problem is formulated as

$$\text{Min}\, F_2(x)$$

subject to

$$g_j(x) \leq 0, \qquad j = 1, 2, \ldots, m$$

and

$$g_{m+1}(x) = F_1(x) - F_1(x_1^*) \leq \epsilon_1 \tag{27}$$

where ϵ_1 is a small value compared to $F_1(x_1^*)$. This procedure is repeated until all k objectives have been considered. The ith problem is given by

$$\text{Min}\, F_i(x)$$

subject to

$$g_j(x) \leq 0, \qquad j = 1, 2, \ldots, m$$

and

$$g_{m+n}(x) = F_n(x) - F_n(x_n^*) \leq \epsilon_n, \qquad n = 1, 2, \ldots, i-1 \tag{28}$$

The solution obtained at the end, i.e., x_k^*, is taken as the desired solution x^* of the multiobjective optimization problem.

The goal programming method was originally proposed by Charnes and Cooper for a linear optimization problem.[15] The method requires goals to be set for each objective that the designer wishes to obtain. A preferred solution is then defined as the one that minimizes the deviations from the set goals. Thus, a simple goal programming problem can be defined as

$$\text{Min}\, F(x) = \left[\sum_{i=1}^{k} (F_i(x))^p \right]^{1/p}, \qquad p \geq 1$$

subject to

$$g_j(x) \leq 0, \qquad j = 1, 2, \ldots, m$$

$$\underline{F}_j(x) \geq 0, \qquad j = 1, 2, \ldots, k$$

where

$$\underline{F}_j(x) = F_j(x) - F_j(x_j^*) \tag{29}$$

Computational Procedure

Analysis

The following analysis procedure is used to study the effects of the variations in the parameters of the structure on the robustness of the system:

1) Start with an initial reference design of the structure and find the corresponding plant matrix A, input matrix B, and output matrix C.
2) Use the LQ regulator design technique to find the optimal control gain G by solving the algebraic Riccati equation.
3) Change the design parameters by known percentage values and find the corresponding A, B, and C matrices.
4) Find the stability robustness index β_{sr}[Eq. (12)] and the performance robustness index β_{pr}[Eq. (17)].
5) Repeat steps 3 and 4 for different parameter changes. (e.g., nominal design variables, damping ratio, density etc.)
6) Plot a graph between β_{sr} or β_{pr} and the change in the parameters.

Design

The purpose of design is to optimize the actively controlled structure by using suitable multiobjective optimization techniques. The procedure is given as follows:

1) From the requirements of stress and deformation, obtain the preliminary design (to be used as the nominal design) of the structure.

2) Construct the plant matrix, input matrix, and output matrix.

3) Formulate the multiobjective constrained optimization problem.

4) Minimize the individual objective functions and find the respective minima around the nominal design.

5) Use a suitable multiobjective optimization approach to find a compromise solution.

Examples

Two-Bar Truss

The two-bar truss shown in Fig. 1 is selected for its simplicity. A nonstructural mass of one unit is attached at node 3. The actuators and the sensors are colocated at node 3, acting in x and y directions. The design variables (cross-sectional areas of the two bars) are restricted to lie between 0.01 and 1.0. The structural damping ratio is considered as 0.01, Young's modulus is assumed to be 10^7, and density is taken to be 4.6. In the performance index, the output weighting matrix Q is assumed to be 1000. I, and the input weighting matrix R is taken to be I, where I is the identity matrix. The natural frequencies of the closed-loop controlled structure are constrained to lie between 20 rad/s and 40 rad/s. For a stable open-loop system, the corresponding feedback closed-loop system must be stable under the optimum control law. But the stability is not guaranteed if there exists disturbances or uncertainties in system parameters. Hence, additional constraints are added on the perturbed closed-loop system, namely, that all the eigenvalues of the perturbed closed-loop system are restricted to have negative real parts.

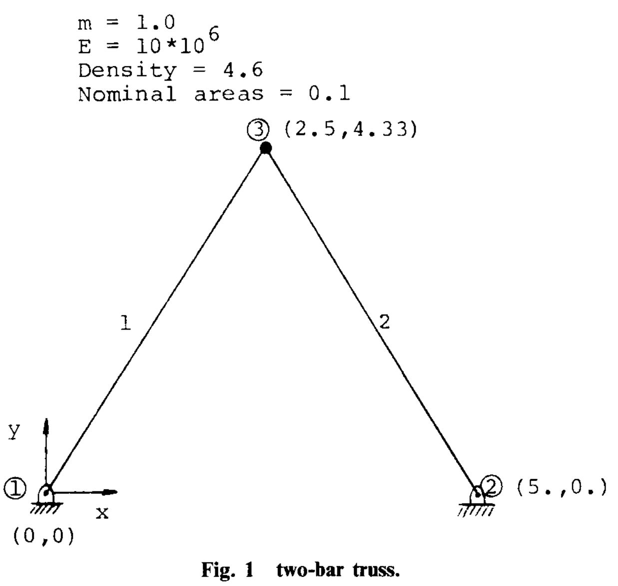


Fig. 1 two-bar truss.

Analysis

Figure 2 shows the relationship between the stability robustness index and the design variables, which can be seen to be a smooth convex function. Figure 3 shows the variation of the performance robustness index with the two design variables. It can be seen to be a nonsmooth function having several local minima in the design space. Figure 4 shows the variations of stability robustness index and performance robustness index with changes in the structural damping ratio of the two-bar truss. It can be seen that the stability robustness index drops sharply at a value of the damping ratio of approximately 4%. The performance robustness index can be observed to attain a minimum value at a damping ratio of approximately 2%. The effect of the variations in Young's modulus of the material on the robustness indices is shown in Fig. 5. It can be seen that the stability robustness index changes very little beyond a value of 5×10^6 of the Young's modulus. The performance robustness index reduces to a minimum value at Young's modulus $(E) = 20 \times 10^6$ and then increases for larger values of E. Figure 6a shows that the system performance index (J) decreases as the damping ratio increases and Fig. 6b indicates that J increases to a maximum value at Young's modulus $E = 20 \times 10^6$, then decreases for larger values of E. The relationship of β_{sr} and β_{pr} with the mass density of the material is shown in Fig. 7. It can be observed that the stability robustness index decreases with an increase in the density of the material. The performance robustness index reduces to a minimum at $\rho = 1.5$ and then increases for higher values of ρ. Figure 8 shows the variations of the stability robustness index and the performance robustness index with a change in the coefficient of the output weighting matrix. An increase in the coefficient of the output weighting matrix implies that the output performance is more important than the control energy. It can be seen that a larger coefficient improves the system stability robustness but reduces the performance robustness of the system. In Fig. 9a, the performance index can be seen to attain a maximum value at $\rho = 1.5$ and then decrease monotonically as material density increases. Figure 9b shows that the performance increases as the coefficient of output weighting matrix increases.

Design

The results of minimization of the individual objective functions are shown in Table 1. The results given by different multiobjective optimization methods are shown in Table 2. The first two columns in Table 2 correspond to formulations 1 and 2 of the utility function method. In formulation 1, w_i

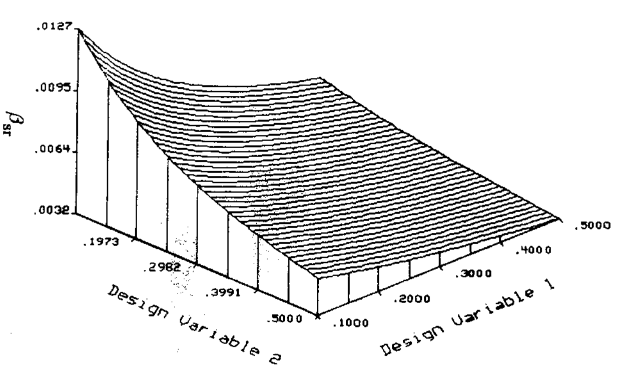


Fig. 2 Stability robustness index vs design variables for two-bar truss.

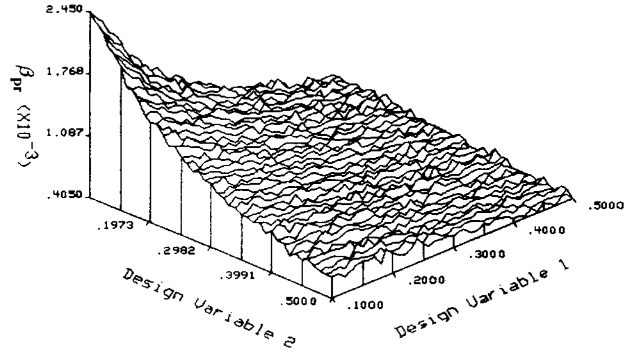


Fig. 3 Performance robustness index vs design variables for two-bar truss.

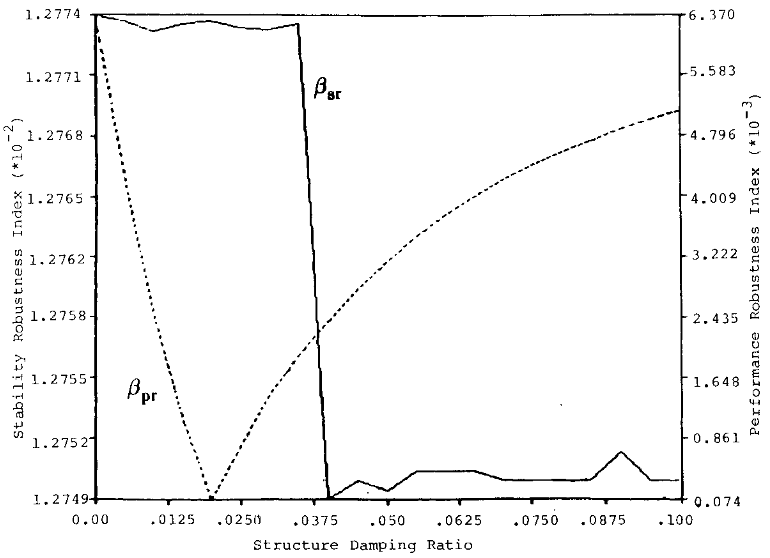


Fig. 4 Variation of β_{sr} and β_{pr} with structural damping ratio for two-bar truss (permissible D.V. change = −5%).

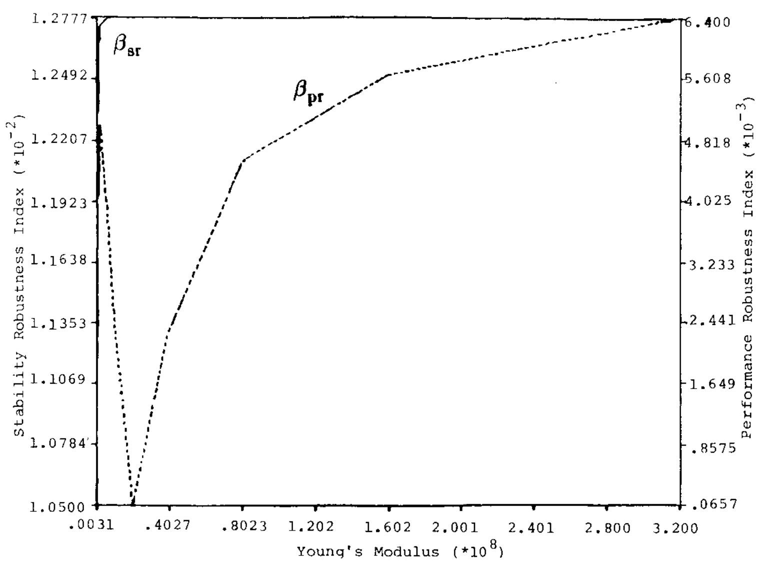
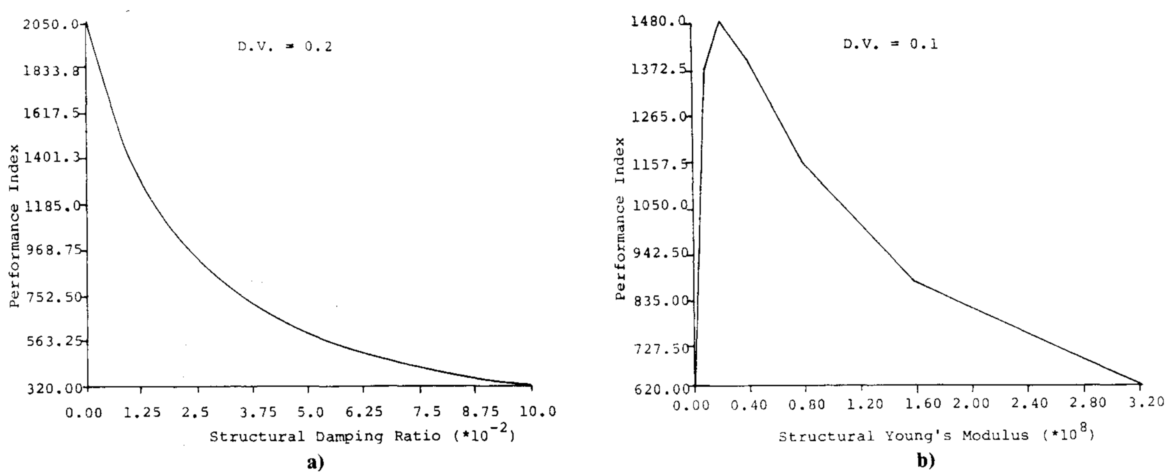


Fig. 5 Variation of β_{sr} and β_{pr} with Young's modulus for two-bar truss (permissible D.V. change = −5%).



a) b)

Fig. 6 Variation of J with structural damping ratio and Young's modulus for two-bar truss.

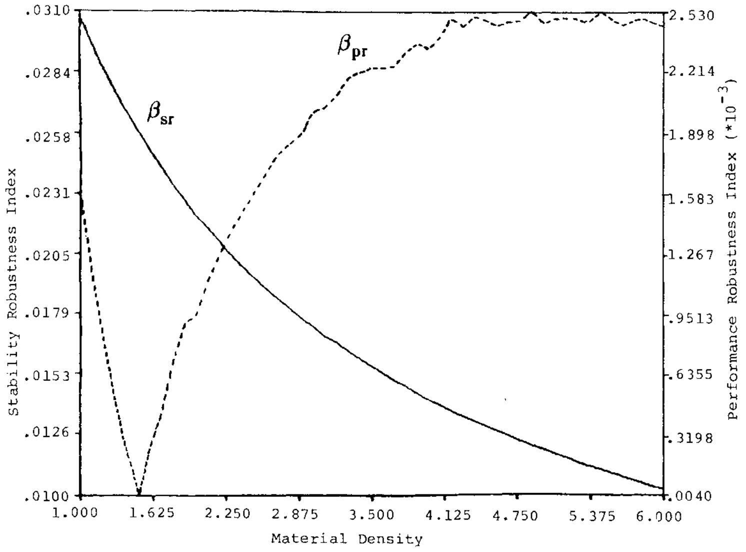


Fig. 7 Variation of β_{sr} and β_{pr} with material density for two-bar truss (permissible D.V. change = −5%).

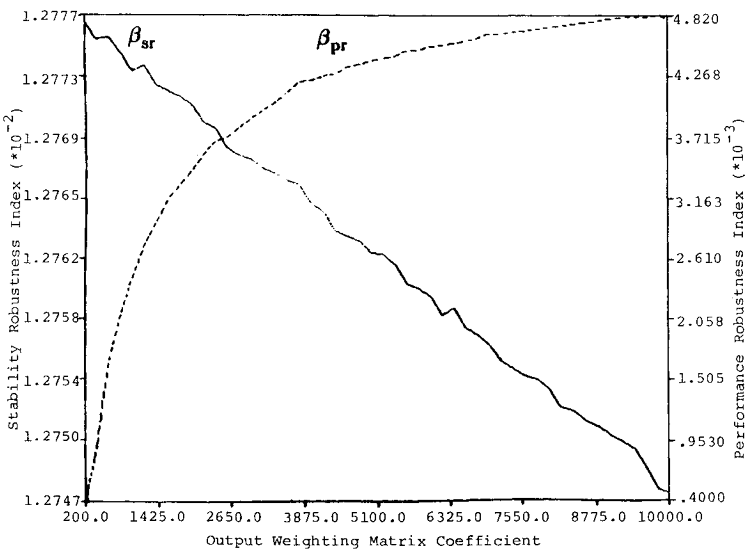


Fig. 8 Variation of β_{sr} and β_{pr} with output wieghting matrix coefficient for two-bar truss (permissible D.V. change = −5%).

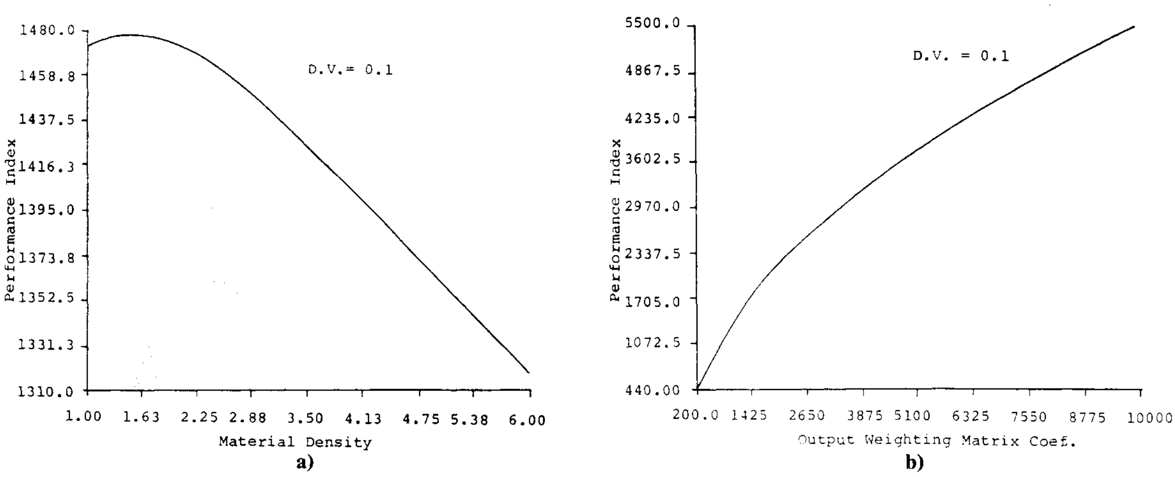


Fig. 9 Variation of J with material density and output weighting matrix coefficient for two-bar truss.

Table 1 Single-objective optimization of two-bar truss. Permissible design variable change = −5%

$$\xi = 0.01, \quad \zeta = 10^3, \quad x(o) = \begin{Bmatrix} 0.1 \\ 0.1 \end{Bmatrix}$$

	Minimization of		
Objective	β_{sr}	β_{pr}	Weight, w
C_i	1.0	0.0	0.0
$i = 1, 3$	0.0	1.0	0.0
	0.0	0.0	1.0
x^*	0.14626	0.15247	0.051301
	0.14626	0.13797	0.051301
$f_1(x^*)$	0.009502	0.009557	0.020037
$f_2(x^*)$	0.001618	0.0015899	0.004813
$f_3(x^*)$	67.28	66.801	23.598
$f^* = \sum_{i=1}^{3} C_i f_i$	0.009502	0.0015899	23.598

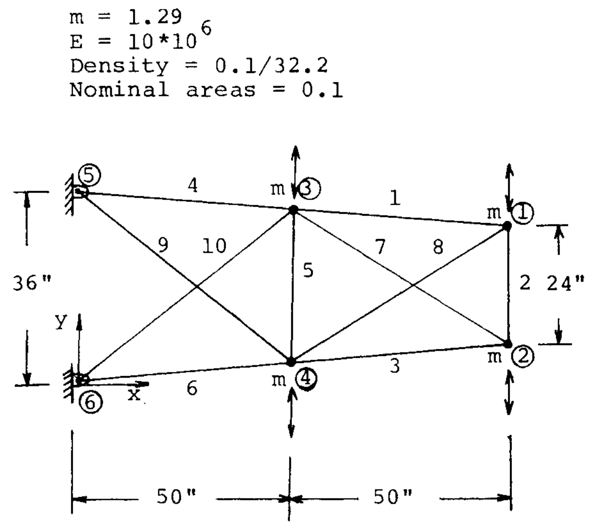


Fig. 10 Two-bay truss.

Table 2 Multiobjective optimization of two-bar truss. Permissible design variable change = −5%

$$\xi = 0.01, \quad \zeta = 10^3, \quad x(o) = \begin{Bmatrix} 0.1 \\ 0.1 \end{Bmatrix}$$

	Utility function method		Lexicographic method			Goal programming method	
			Optimization order				
	Const. coef.	Variable coef.	f_1, f_2, f_3	f_3, f_1, f_2	f_2, f_1, f_3	$p = 1$	$p = 2$
Optimal design variables X^*	$X_1 = 0.1309$	0.1463	0.15389	0.056296	0.14466	0.051294	0.051807
	$X_2 = 0.12709$	0.1299	0.13483	0.056303	0.1440	0.051309	0.051807
$f_1(X^*)$	0.010506	0.009950	0.009603	0.018932	0.009604	0.020037	0.019919
$f_2(X^*)$	0.001867	0.001792	0.001669	0.004399	0.001664	0.004809	0.004770
$f_3(X^*)$	59.337	63.549	68.404	25.898	66.391	23.599	23.831
$\sum_{j=1}^{3} F_i(X^*)$	0.999438	0.977291	1.014075	1.819317	1.010398	1.998782	1.980792

are set equal to a fixed value of 1/3 in Eq. (34); w_i are considered as design variables in formulation 2. The last row of Table 2 gives the values of the global evaluation function, which can be used as an index to compare the results of different multiobjective optimization methods. The global evaluation function is defined as

$$F_g(x) = \sum_{i=1}^{3} F_i(x^*)$$

where

$$F_1(x) = \frac{[f_1(x) - 0.009502]}{0.010535}$$

$$F_2(x) = \frac{[f_2(x) - 0.0015899]}{0.0032231}$$

and

$$F_3(x) = \frac{[f_3(x) - 23.598]}{43.682} \tag{30}$$

Two-Bay Truss

The finite-element model of the second example (two-bay truss) is shown in Fig. 10. For this example, nonstructural masses of magnitude 1.29 are attached at nodes 1–4. Each node has two degrees of freedom. The actuators and sensors are colocated at nodes 1–4 and are assumed to act along the y direction only. The design variables (cross-sectional areas) are restricted to lie between 0.001 and 0.5 The natural frequencies of the closed-loop system are constrained to be larger than 31.62 rad/s (i.e., $\omega^2 \geq 1000$).

Analysis

This example has ten design variables. Since the display of functional relations in ten-dimensional design space is not possible, the variation of the robustness of the system is found by uniformly varying the value of all the ten design variables. The results are shown in Fig. 11. This figure shows the stability robustness index vs the value of the design variables when the permissible change in the design vector is assumed to be −5%. It can be observed that β_{sr} decreases slowly with an increase in the value of the design variables, but it is not a convex function. Hence, local minima are expected in the optimization process; i.e., the optimum solution will depend on the initial guess. Also, β_{pr} decreases until a value of 0.2 for the design vector, and then increases. This relationship is also not a convex function, as in the case of β_{sr}. In Fig. 12, the performance index can be seen to decrease monotonically with the increase in the value of the design vector. This implies that a stronger structure will induce a smaller displacement and need lesser control energy to obtain good performance.

Design

The nominal value of the design variables are assumed to be $x_i = 0.1$, $i = 1-10$. Table 3 gives the results obtained by optimizing the individual objective functions, starting from the nominal design. The results of different multiobjective

Table 3 Single-objective optimization of two-bay truss. Permissible design variable change = −5%

$\xi = 0.01$, $\zeta = 10^3$, $X_i(o) = 0.1$, $i = 1–10$

	Minimization of		
Objective	β_{sr}	β_{pr}	Weight, w
	0.13816	0.13765	0.11274
	0.09648	0.09339	0.00100
Optimal	0.13782	0.13777	0.11315
design	0.27661	0.27637	0.33788
variables	0.10103	0.09899	0.00100
X_i^*	0.27780	0.27725	0.33772
$i = 1–10$	0.14537	0.14671	0.11810
	0.14417	0.14507	0.11884
	0.14808	0.14998	0.12110
	0.15119	0.14920	0.12122
$f_1(X^*)$	0.048694	0.048701	0.049354
$f_2(X^*)$	0.010019	0.00981975	0.0084567
$f_3(X^*)$	0.252557	0.252307	0.22730

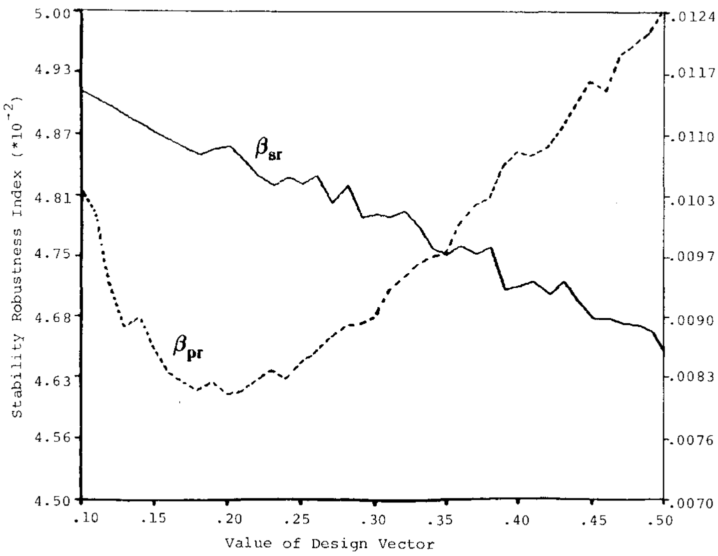


Fig. 11 Variation of β_{sr} and β_{pr} with design variables for two two-bay truss (permissible D.V. change = −5%).

Table 4 Multiobjective optimization of two-bay truss. Permissible design variable change = −5%

$\xi = 0.01$, $\zeta = 10^3$, $X_i(o) = 0.1$, $i = 1, 10$

Approach	Utility function method		Lexicographic method		Goal programming method
	Const. coef.	Variable coef.	Optimization order		$p = 2$
			f_1, f_2, f_3	f_3, f_1, f_2	
	0.12247	0.13753	0.13697	0.14197	0.11389
	0.01779	0.09207	0.09588	0.03602	0.00100
Optimal	0.11406	0.13603	0.15071	0.16162	0.11142
design	0.32120	0.27907	0.29918	0.36978	0.00100
variables	0.01000	0.09880	0.07856	0.00100	0.00100
X_i^*	0.33931	0.27920	0.29422	0.34327	0.33617
$i = 1,10$	0.10835	0.14554	0.15018	0.11642	0.11907
	0.12838	0.14278	0.15061	0.12668	0.11901
	0.13045	0.14917	0.10706	0.10488	0.12277
	0.11834	0.14972	0.12839	0.11305	0.12420
$f_1(X^*)$	0.048769	0.048742	0.048788	0.048664	0.049360
$f_2(X^*)$	0.012034	0.00997	0.010307	0.011721	0.008479
$f_3(X^*)$	0.22903	0.25196	0.24841	0.24438	0.22736
$\sum_{i=1}^{3} F_i(X^*)$	2.431212	1.946322	2.094988	2.694381	0.999241

optimization methods, namely, the utility function method, the lexicographic method, and the goal programming method, are compared in Table 4. The last row of Table 4 shows the global evaluation function F_g defined as

$$F_g(x) = \sum_{i=1}^{3} F_i(x^*)$$

where

$$F_1(x) = \frac{[F_1(x) - 0.048694]}{0.000668}$$

$$F_2(x) = \frac{[f_2(x) - 0.008454]}{0.001588}$$

and

$$F_3(x) = \frac{[f_3(x) - 0.2273]}{0.02681} \quad (31)$$

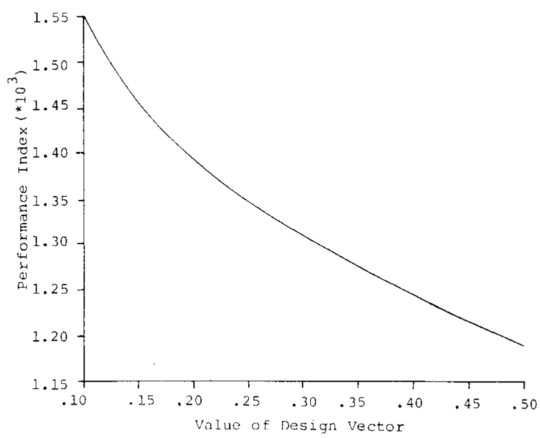


Fig. 12 Variation of J with design variables for two-bay truss.

Summary and Conclusions

The stability robustness index and the performance robustness index defined in this work have been found to be highly nonlinear with respect to design variables. The nonconvex property of robustness indices, with changes in design variables, leads to difficulties in optimization. As such, one can expect only a local minimum in the neighborhood of the starting design during optimization. In general, the local optima are acceptable, since the starting design is usually taken as the nominal design, which is expected to be robust. Techniques, such as hill jumping and annealing, can be applied to escape the local minima to obtain the global optimal design. The relationships between the stability/performance robustness index and the various system parameters have been determined numerically for the two-bar truss. These results are expected to be useful in choosing suitable material for a given structure, with a specified geometry or weighting coefficients in the performance index for controller design.

A major advantage of using nonlinear programming to find the robust control/structural design is that it can be used with large permissible changes in the design variables and/or different constraint specifications. Three multiobjective optimization methods have been used to find the optimal designs of the illustrative examples. For the two-bar truss, the utility function method with variable coefficients gave the smallest value of the global evaluation function. For the two-bay truss, the goal programming method, with $p = 2$, yielded the smallest value for the global evaluation function, and the utility function method with variable coefficients gave the second smallest value. As observed in the investigation, no particular method gives the best solution for all the problems. Hence, several methods are to be used to solve the problem and find the best tradeoff between the multiple objectives.

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